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ON Soft πGR-Separation Axioms in Soft Topological Spaces

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Abstract
The aim of this paper is to define soft πgr-separation axioms and obtain some of their characterizations.

Keywords: soft πgr-closed set, soft πgr-continuous ,soft πgr-irresolute, soft M-πgr-open map, soft πgr-T_i-space(i=0,1,2), soft πgr-regular space ,soft πgr-normal space.

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Introduction

Levine[10] introduced g-closed sets in general topology. The concept of π-closed sets in topological spaces was initiated by Zaitsav[17] and the concept of πg-closed set was introduced by Noiri and Dontchev[4]. N.Palaniappan[14] studied and introduced regular closed sets in topological spaces. Janaki.C and Jeyanthi.V [8] introduced πgr-closed sets in topological spaces.

Molodtsov[13] introduced the concept of soft set as a new mathematical tool. Maji et.al[12] gave first practical application of soft sets in decision making problems. D.N.Georgiou and A.C. Megaritis[5] introduced soft set theory and soft topology and studied properties for mappings between different classes of soft sets . Soft separation axioms for soft topological space were studied and introduced by Shabir et al[15]. Arokia Lancy and Arokiarani[1] introduced soft β-separation axioms and derived some of its characteristics. Soft semi-open sets and its properties along with soft semi-separation axioms were introduced and studied by Bin Chen[2]. Introduction of soft point alongwith the study on interior point, neighborhood system, continuity and compactness is found in [18] due to Zorlutuna.On soft topological space via semi open and semi closed soft sets was introduced and studied by J. Mahanta and P.K. Das [11]. Kharal et al.[9]introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [3] in 2013 studied and discussed the properties of soft sontinuous mappings . In 2013, Janaki.C and Jeyanthi.V[6] introduced soft πgr-closed sets [9] in soft topological spaces.

In this paper, we introduce and study soft πgr-T_i(i=0,1,2)-spaces ,soft πgr-regular and soft πgr -normal spaces.

Preliminaries

Definition 2.1 [5,11,16]
Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For Ɛ∈A, F(Ɛ) may be considered as the set Ɛ- approximate elements of the soft set (F,A).Clearly, a soft set is not a set.
Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition:2.2[5,11,16]
For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U− F(e) for all e∈A.

Definition :2.3[16,11]
A Soft set (F,A) over X is said to be a Null soft set denoted by $\bar{\varphi}$ or φ_A if for all e∈A, F(e)=φ(null set).

Definition:2.4[5,11,16]

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A soft set (F,A) over X is said to be absolute soft set denoted by $\bar{A}$ or X_A if for all e∈A, F(e) =X. Clearly , we have $X_A' = \varphi_A$ and $\varphi_A' = X_A$.

Definition :2.5 [5,11,16]

The union of two soft sets of (F,A) and (G,B) over the common universe U is soft set (H,C), where C = A∪B and for all e∈C, H(e) = F(e) if e ∈A−B, H(e) = G(e) if e∈B−A and H(e)=F(e) ∪G(e) if e ∈A∩B. We write (F,A) ∪ (G,B)= (H,C).

The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A)∩(G,B) is defined as C= A∩B and H(e) = F(e) ∩G(e) for all e∈C.

Definition:2.6

Let (F,E) be a soft set over X and x ∈ X. We say that x ∈ (F,E) read as x belongs to the soft set (F,E), whenever x ∈ F(e) for all e ∈ E.

Note that for x ∈ X, x ∉ (F,E) if x ∉ F(e) for some e ∈ E.

Definition:2.7 [5,11,16]

Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

1) φ, $\widetilde{X}$ belong to τ.
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Let us denote the collection of soft sets over the universe X and Y as SS(X) and SS(Y) respectively.

Definition:2.8 [5,11,16]

Let (X, τ, E) be soft topological space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ.

Definition:2.9 [11,16]

Let (X,τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F,E) is smallest soft closed set containing (F,E).

Int (F,E) =∪{ (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)}.

Cl(F,E) = ∩{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition:2.10

A soft subset (A,E) of (X,τ,E) is called

(i) a soft semi open [2,10]if (A,E) $\tilde{\subseteq}$ Int(Cl(A,E))

(ii))a soft regular open[6] if (A,E)= Int(Cl(A,E)).

(iii) a soft clopen is (A,E) is both soft open and soft closed.

The complement of the soft semi open , soft regular open subsets are their respective soft semi closed , soft regular closed subsets .

The finite union of soft regular open sets is called soft π-open set and its complement is soft π-closed set.

Definition:2.11[6]

Let (X,τ, E) be a soft topological space over X and the soft regular interior of (F,E) denoted by srint(F,E) is the union of all soft regular open subsets of (F,E).The soft regular closure of (F,E) denoted by srcl(F,E) is the intersection of all regular closed sets containing (F,E).

srint (F,E) =∪{ (O,E): (O,E) is soft regular open and (O,E) $\tilde{\subseteq}$ (F,E)}.

srcl(F,E) = ∩{ (O,E): (O,E) is soft regular closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition:2.12[11]

Let (F,E) be a soft set over (X,τ, E). The soft set (F,E) is called a soft point denoted by e_F if for the element e∈E , F(e)≠φ and F(e′)=φ for all e′∈E-{e}.

Definition:2.13 ([9],[11])

Let $SS(X)_A$ and $SS(Y)_B$ be two soft classes,where A,B ∈ E. Then u: X→Y and p:A→B be two functions. Then the function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ and its inverse are defined as

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(i) Let (F,A) be a soft set in $SS(X)_A$. The image of (F,A) under f_{pu}, written as $f_{pu}((F,A)) = (f_{pu}(F),p(A))$is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(y) = \begin{cases} \left(\bigcup_{x\in p^{-1}(y)\cap A} u(F(x))\right), & p^{-1}(y)\cap A\neq\phi \\ \phi, & \text{otherwise.} \end{cases}$$

for all y∈B.

(ii) Let (G,B) be a soft set in $SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}((G,B))= (f_{pu}^{-1}(G),p^{-1}(B))$ is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(x)= \begin{cases} u^{-1}(G(p(x))), & p(x)\in B, \\ \phi, & \text{otherwise.} \end{cases}$$

for all x∈A.

Definition:2.14([11])

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces and $f_{pu} : SS(X)_A\to SS(Y)_B$ be a function. Then function f_{pu} is called soft continuous if $f_{pu}^{-1}((G,B)) \in \tau$ for all (G,B)∈τ*.

Definition:2.15 [11]

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces, where A,B∈E and $f_{pu}:SS(X)_A\to SS(Y)_B$ be a function. Then the function f_{pu} is called soft open mapping if $f_{pu}((G,A)) \in \tau^*$ for all (G,A)∈τ.

Similarly, a function $f_{pu}:SS(X)_A\to SS(Y)_B$ is called a soft closed map if for a closed set (F,A) in τ, the image $f_{pu}((G,B))$ is soft closed in τ*.

Definition:2.16([6])

A soft subset (G,E) of a soft topological space X is called a soft πgr-closed set in X if srcl(G,E) $\tilde{\subset}$ (X,E)) whenever (G,E) $\tilde{\subset}$ (X,E), where (X,E) is soft π- open in X. We denote the soft πgr-closed set of X by SπGRC(X) . The complement of soft πgr-closed set is soft πgr-open set and is denoted by SπGRO(X).

Definition:2.17[6]

The Soft πgr-Closure of a soft set (G,E) is defined to be the intersection of all soft πgr-closed sets containing the soft set (G,E) and is denoted by sπgr-cl(G,E).

The Soft πgr-Interior of a soft set (G,E) is defined to be the union of all soft πgr-open sets contained the soft set (G,E) and is denoted by sπgr-int(G,E).

Let us take the function f_{pu} as simply f.

Definition:2.18[7]

A function f: $SS(X)_A\to SS(Y)_B$ (where A ,B ∈ E) is

(i) Soft πgr-continuous if $f^{-1}((G,B))$ is soft πgr-closed in X for every soft closed set (G,B) of Y.

(ii) Soft πgr-irresolute if $f^{-1}((G,B))$ is soft πgr-closed in X for every soft πgr-closed set (G,B) of Y.

Definition:2.19

A function f: $SS(X)_A\to SS(Y)_B$ (where A ,B ∈ E) is

(i) Soft M-πgr-open if f((F,A)) is soft πgr-open in Y for every soft πgr -open set (F,A) of X.

(ii) Soft regular open(soft regular closed) if f((F,A)) is soft regular open (soft regular closed) in Y for every soft open set (soft closed set)(F,A) of X.

Definition: 2.20[11]

A soft topological space (X,τ,E) is said to be soft T_0-space if for two disjoint points x and y ,there exists a soft open sets (F,E) and (G,E) containing one but not the other.

Definition: 2.21[11]

A soft topological space (X,τ,E) is said to be soft T_1-space if for two disjoint points x and y of X, there exists a soft open sets (F,E) and (G,E) such that x $\tilde{\in}$ (F,E) , y $\tilde{\notin}$ (F,E) and y $\tilde{\in}$ (G,E) and x $\tilde{\notin}$ (G,E)

Definition: 2.22[11]

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A soft topological space (X,τ,E) is said to be soft T_2-space if for two disjoint points x and y of X, there exists a disjoint soft open sets (F,E) and (G,E) such that $x \tilde{\in}$ (F,E) and $y \tilde{\in}$ (G,E) .

Definition:2.23[11]

A soft topological space (X,τ,E) is said to be soft regular space if for every $x \in X$ and a soft closed set (F,E) not containing x, there exists soft open sets (G,E) and (H,E) such that $x \tilde{\in}$ (G,E) , (F,E) $\tilde{\in}$ (H,E) and (G,E) $\cap$ (H,E)= ϕ .

Definition:2.24[11]

A soft topological space (X,τ,E) is said to be soft normal space if for every pair of disjoint soft closed sets (F_1,E) and (F_2,E),there exists disjoint soft open sets (G_1,E) and (G_2,E) such that (F_1,E) $\tilde{\subseteq}$ (G_1,E) and (F_2,E) $\tilde{\in}$ (G_2,E)

SOFT πGR- SEPARATION AXIOMS.

Definition:3.1

A soft topological space (X,τ,E) is said to be soft πgr -T_0-space if for two disjoint points x and y of X,there exists a soft πgr -open sets (F,E) and (G,E) containing one but not the other.

Theorem:3.2

A soft space X is soft -πgr-T_o -space iff soft πgr-closures of distinct points are distinct.

Proof: Let x and y be distinct points of X. Since X is a soft πgr-T_o-space, there exists a soft πgr-open set (F,E) such that $x \tilde{\in}$ (F,E) and $y \tilde{\notin}$ (F,E).

Consequently , X−(F,E) is a soft πgr-closed set containing y but not x. But s-πgr-cl(y) is the intersection of all soft πgr-closed sets containing y. Hence $y \tilde{\in}$ s-πgr-cl(y), but $x \tilde{\notin}$ s-πgr-cl(y) as $x \tilde{\notin}$ X−(F,E).Therefore, s-πgr-cl(x) ≠s-πgr-cl(y).

Conversely, let s-πgr-cl(x) ≠s-πgr-cl(y) for x≠y.

Then there exists atleast one point $z \in X$ such that $z \tilde{\notin}$ s-πgr-cl(y).

We have to prove $x \tilde{\notin}$ s-πgr-cl(y), because if $x \tilde{\in}$ s-πgr-cl(y), then {x} $\tilde{\subseteq}$ s- πgr-cl(y)

$\Rightarrow$ s-πgr-cl(x) $\tilde{\subseteq}$ s-πgr-cl(y).So, z $\tilde{\in}$ s-πgr-cl(y), which is a contradiction. Hence $x \tilde{\notin}$ s-πgr-cl(y) $\Rightarrow x \tilde{\in}$ X – s-πgr-cl(y), which is a soft πgr-open set containing x but not y. Hence X is a soft πgr-T_o-space.

Definition:3.3

A soft topological space (X,τ,E) is said to be soft πgr -T_1-space if for two disjoint points x and y of X, there exists a soft πgr -open sets (F,E) and (G,E) such that $x \tilde{\in}$ (F,E) , $y \tilde{\notin}$ (F,E) and $y \tilde{\in}$ (G,E) and $x \tilde{\notin}$ (G,E)

Theorem:3.4

A soft space X is soft πgr-T_1-space iff each singleton is soft πgr-closed set.

Proof: Let X be a soft πgr-T_1-space and x∈X .Let y ∈X−{x}. Then for x≠y, there exists πgr-open set U_y such that y∈U_y and x∉U_y.

Conversely, y∈U_y ⊂X-{x}.

That is X−{x} = $\cup\{U_y$: y∈X−{x}},which is soft πgr-open set.

Hence {x} is soft πgr-closed set.

Conversely, suppose {x} is soft πgr-closed set for every x∈X. Let x,y ∈X with x≠y. Now, x≠y⇒y∈X−{x}. Hence X−{x} is soft πgr-open set containing y but not x. Similarly, X−{y} is soft πgr-open set containing x but not y. Therefore, X is a soft πgr-T_1-space.

Theorem:3.5

If every point (x,E) of a soft topological space (X,τ,E) is a soft πgr-closed , then (X,τ,E) is a soft πgr-T_1-space.

Proof: Let (x,E) and (y,E) be two distinct points of (X,τ,E) and by hypothesis (x,E) and (y,E) are soft πgr-closed .Then (x,E)′ and (y,E)′ are soft πgr-open such that (y,E) $\tilde{\in}$ (x,E)′ , (y,E) $\tilde{\notin}$ (x,E) and (x,E) $\tilde{\in}$ (y,E)′ , (x,E) $\tilde{\notin}$ (y,E) .

Hence the soft topological space (X,τ,E) is a soft πgr-T_1-space.

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Theorem:3.6

If $f : SS(X)_E \to SS(Y)_E$ is soft M- πgr –open soft bijective map and X is soft πgr -T_1 -space, then Y is soft πgr- T_1-space.

Proof: Let $f : SS(X)_E \to SS(Y)_E$ be soft bijective and soft M-πgr-open function. Let X be a soft πgr-T_1-space and y_1,y_2 be any two distinct points of Y.

Since f is soft bijective, there exists distinct points x_1, x_2 of the soft space X such that $y_1=f(x_1)$ and $y_2=f(x_2)$, where y_1 and y_2 are distinct points of the soft space Y. Now, X being a soft πgr-T_1-space, there exists soft πgr-open sets (G,E) and (H,E) such that $x_1 \in (G,E), x_2 \notin (G,E)$ and $x_1 \notin (H,E)$, $x_2 \in (H,E)$. Since $y_1=f(x_1) \in (G,E))$ but $y_2=f(x_2) \notin f((G,E))$ and $y_2= f(x_2) \in f((H,E))$ and $y_1 = f(x_1) \notin f((H,E))$.

Now, f being soft M-πgr-open, f((G,E)) and f((H,E)) are soft πgr-open sets of Y such that $y_1 \in f((G,E))$ but $y_2 \notin f((G,E))$ and $y_2 \in f((H,E))$ and $y_1 \notin f((H,E))$. Hence Y is soft πgr-T_1-space.

Theorem :3.7

If $f : SS(X)_A \to SS(Y)_B$ is soft πgr -continuous injection and Y is soft T_1-space , then X is πgr- T_1- space.

Proof: Let $f:SS(X)_A \to SS(Y)_B$ be a soft πgr- continuous injection and Y be soft T_1 . For any two distinct points x_1, x_2 of the soft space X, there exists distinct points y_1,y_2 of the soft space Y such that $y_1 = f(x_1)$ and $y_2= f(x_2)$.

Since Y is soft T_1- space, there exists soft open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \notin (F,E)$ and $y_1 \notin (G,E)$, $y_2 \in (G,E)$.

i.e. $x_1 \in f^{-1}((F,E)), x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E)), x_2 \notin f^{-1}((F,E))$

Since f is soft πgr - continuous, $f^{-1}((F,E)), f^{-1}((G,E))$ are soft πgr -open sets in X.

Thus for two distinct points x_1,x_2 of the soft space X, there exists soft πgr - open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E)), x_2 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E)), x_2 \notin f^{-1}((F,E))$.

Therefore, X is a soft πgr -T_1 - space.

Theorem :3.8

If $f : SS(X)_E \to SS(Y)_B$ be soft πgr –irresolute function, and Y is soft πgr - T_1 - space, then X is soft πgr -T_1 -space.

Proof: Let x_1, x_2 be two distinct points in a soft space X. Since f is injective, there exists distinct points y_1, y_2 of a soft space Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$.

Since Y is soft πgr -T_1 -space, there exists soft πgr- open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \notin (F,E)$ and $y_1 \notin (G,E)$, $y_2 \in (G,E)$.

i.e. $x_1 \in f^{-1}((F,E)), x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E)), x_2 \notin f^{-1}((F,E))$.

Since f is soft πgr- irresolute, $f^{-1}((F,E))$, $f^{-1}((G,E))$ are soft πgr - open sets in X.

Thus, for two distinct points x_1,x_2 of X, there exists soft πgr- open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E)), x_1 \notin f^{-1}((G,E))$ and $x_2 \in f^{-1}((G,E)), x_2 \notin f^{-1}((F,E))$.

Hence X is a soft πgr - T_1 - space.

Definition:3.9

A soft topological space (X,τ,E) is said to be soft πgr -T_2-space if for two disjoint points x and y of X, there exists a disjoint soft πgr- open sets (F,E) and (G,E) such that $x \tilde{\in} (F,E)$ and $y \tilde{\in} (G,E)$.

Theorem : 3.10

If $f : SS(X)_E \to SS(Y)_B$ be soft πgr -continuous injection, and Y is soft T_2-space, then X is soft πgr -T_2 -space.

Proof: Let $f : SS(X)_E \to SS(Y)_E$ be soft πgr- continuous injection and Y be soft-T_2-space.Then for any two distinct points x_1 and x_2 of a soft space X, there exists distinct points y_1, y_2 of a soft space Y such that $y_1 = f(x_1)$, $y_2 = f(x_2)$.

Since Y is soft T_2 -space, there exists disjoint soft open sets (F,E) and (G,E) in Y such that $y_1 \in (F,E)$ and $y_2 \in (G,E)$.

i.e. $x_1 \in f^{-1}((F,E)), x_2 \in f^{-1}((G,E))$.

Since f is soft πgr - continuous, $f^{-1}((F,E))$ & $f^{-1}((G,E))$ are soft πgr - open sets in X.

Further f is soft injective, $f^{-1}((F,E)) \cap f^{-1}((G,E)) = f^{-1}((F,E)\cap(G,E)) = f^{-1}(\phi)= \phi$.

Thus, for two disjoint points x_1, x_2 of X, there exists disjoint soft πgr-open sets $f^{-1}((F,E))$ and $f^{-1}((G,E))$ such that $x_1 \in f^{-1}((F,E))$ and $x_2 \in f^{-1}((G,E))$.Hence X is soft πgr-T_2-space.

Theorem :3.11

If $f : SS(X)_E \to SS(Y)_E$ be the soft πgr irresolute injective function and Y is soft πgr- T_2 -space, then X is soft πgr-T_2-space.

Proof : Let x_1,x_2 be any two distinct points in a soft space X. Since f is soft injective, there exists distinct points y_1,y_2 of a soft space Y such that $y_1= f(x_1)$ and $y_2 = f(x_2)$.

Since Y is soft πgr-T_2, there exist disjoint soft πgr-open sets (F,E) and (G,E) in Y such that $y_1 \in$(F,E) and $y_2 \in$(G,E).

i.e, $x_1 \in f^{-1}$((F,E)), $x_2 \in f^{-1}$((G,E)).

Since f is soft πgr-irresolute , f^{-1}((F,E)), f^{-1}((G,E)) are disjoint soft πgr-open sets in X.

Thus, for two distinct points x_1,x_2 of X, there exists disjoint soft πgr-open sets f^{-1}((F,E)) and f^{-1}((G,E)) such that $x_1 \in f^{-1}$((F,E)) and $x_2 \in f^{-1}$((G,E)).

Hence X is a soft πgr-T_2-space.

Theorem:3.12

A soft topological space (X,τ,E) is soft πgr-T_2 iff for distinct points x,y of X, there exists a soft πgr-open set (F,E) containing x but not y such that y $\tilde{\notin}$ s-πgr-cl(F,E)

Proof:

Let x and y be two distinct points in a soft πgr-T_2-space (X,τ,E). Then there exists distinct soft πgr-open sets (G,E) and (H,E) such that x $\tilde{\in}$ (G,E) and y $\tilde{\in}$ (H,E) . This implies x $\tilde{\in}$ (H,E)′. So, (H,E)′ =(F,E) is soft πgr-closed set containing x but not y and s-πgr-cl(F,E)=(F,E).

Hence y $\tilde{\notin}$ s-πgr-cl(F,E).

On the other hand, let x and y be two distinct points of (X,τ,E). Then there exists a soft πgr-open set (F,E) containing x but not y such that y $\tilde{\notin}$ s-πgr-cl(F,E).

$\Rightarrow$ y $\tilde{\in}$ (s-πgr-cl(F,E))′.

Hence (F,E) and (s-πgr-cl(F,E))′ are disjoint soft πgr- open sets containing x and y respectively.

$\Rightarrow$ The space (X,τ,E) is a soft πgr-T_2-space.

Definition:3.13

A soft topological space (X,τ,E) is said to be soft πgr -regular space if for every $x \in X$ and a soft πgr -closed set (F,E) not containing x, there exists soft πgr -open sets (G,E) and (H,E) such that x $\tilde{\in}$ (G,E) , (F,E) $\tilde{\in}$ (H,E) and (G,E) $\cap$ (H,E)= ϕ .

Theorem: 3.14

Every soft πgr-regular and soft T_o - space is soft πgr-T_2.

Proof : Let x,y $\in$ X such that x≠y.

Let X be a soft T_o-space and (G,E) be a soft open set which contains x but not y.

Then X-(G,E) is a soft closed set containing y but not x. Now, by soft πgr-regularity of X, there exists disjoint soft πgr-open sets (A,E) and (B,E) such that x $\in$ (A,E) and X−(G,E)⊂(B,E).

Since y∈X−(G,E), y∈(B,E).

Thus, for x,y $\in$ X with x≠y ,there exists disjoint soft πgr-open sets (A,E) and (B,E) such that x∈ (A,E) and y ∈(B,E).

Hence X is soft πgr-T_2-space.

Theorem :3.15

If f: $SS(X)_E \rightarrow SS(Y)_E$ is soft continuous,bijective ,πgr- open function and X is a soft regular space, then Y is soft πgr-regular.

Proof: Let (F,E) be a soft closed set in Y and let y be a point in a soft space Y in which y∉(F,E).

Take y=f(x) for some point x in a soft space X.

Since f is soft continuous, f^{-1}((F,E)) is soft closed set in X such that x∉ f^{-1}((F,E)). (since f(x)∉(F,E))

Now, X is soft regular, there exists disjoint soft open sets (A,E) and (B,E) such that x∈(A,E) and f^{-1}((F,E))⊂(B,E).

i.e. y=f(x) $\in$ f((A,E)) and (F,E)⊂ f((B,E)).

Since f is soft πgr-open function, f((A,E)) and f((B,E)) are soft πgr-open sets in Y.

Since f is soft bijective, f((A,E)) ∩ f((B,E)) = f((A,E)∩(B,E))=f(ϕ)= ϕ.

⇒ Y is soft πgr-regular.

Theorem:3.16

If f: $SS(X)_E \rightarrow SS(Y)_E$ is soft continuous, soft bijective,soft M- πgr-open function and X is soft πgr-regular space, then Y is soft πgr-regular .

Proof : Let (F,E) be a soft closed set in Y and y $\tilde{\notin}$ (F,E).

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Take y=f(x) for some point x in a soft space X.
Since f is soft continuous and soft bijective, $f^{-1}((F,E))$ is soft closed in X and x $\tilde{\notin}$ $f^{-1}((F,E))$.
Now, since X is soft πgr-regular, there exists disjoint soft πgr-open sets (A,E) and (B,E) such that x ∈(A,E) and $f^{-1}((F,E))\subset(B,E)$.
i.e. y=f(x) ∈f((A,E)) and (F,E)⊂f((B,E)).
Since f is soft M-πgr-open and soft bijective, f((A,E)) and f((B,E)) are disjoint soft πgr-open sets in Y. Therefore,Y is soft πgr-regular .

Theorem :3.17
If $f:SS(X)_E\rightarrow SS(Y)_E$ is soft πgr-continuous, soft closed injection map and Y is soft regular, then X is soft πgr-regular.
Proof: Let (F,E) be a soft closed set in X and x $\tilde{\notin}$ (F,E).
Since f is soft closed injection, f((F,E)) is soft closed set in Y such that f(x) $\tilde{\notin}$ f((F,E)).
Now, Y is soft regular, there exists disjoint soft open sets (G,E) and (H,E) such that f(x) ∈ (G,E) and f((F,E))⊂ (H,E).
This implies x∈$f^{-1}((G,E))$ and (F,E) $\tilde{\subseteq}$ $f^{-1}((H,E))$.
Since f is soft πgr-continuous, $f^{-1}((G,E))$ and $f^{-1}((H,E))$ are soft πgr-open sets in X.
Further, $f^{-1}((G,E)) \cap f^{-1}((H,E)) = \phi$.
Hence X is soft πgr-regular.
Theorem :3.18
If f: $SS(X)_E\rightarrow SS(Y)_E$ is soft πgr-irresolute, soft closed injection and Y is soft πgr-regular,then X is soft πgr-regular.
Proof: Let (F,E) be a soft closed set in X and x∉(F,E).Since f is soft closed injection, f((F,E)) is soft closed set in Y such that f(x) $\tilde{\notin}$ f((F,E)).
Now, Y is soft πgr-regular, there exists disjoint soft πgr-open sets (G,E) and (H,E) such that f(x)∈(G,E) and f((F,E)) $\tilde{\subseteq}$ (H,E).
⇒ x $\tilde{\in}$ $f^{-1}((G,E))$ & F $\tilde{\subseteq}$ $f^{-1}((H,E))$.
Since X is soft πgr-irresolute, f^{-1} ((G,E)) and $f^{-1}((H,E))$ are soft πgr-open sets in X.
Further, $f^{-1}((G,E)) \cap f^{-1}((H,E)) = \phi$ and hence X is soft πgr-regular.
Definition:3.19
A soft topological space (X,τ,E) is said to be soft πgr -normal space if for every pair of disjoint soft πgr- closed sets (F_1,E) and (F_2,E),there exists disjoint soft πgr -open sets (G_1,E) and (G_2,E) such that $(F_1,E)\tilde{\subseteq}(G_1,E)$ and $(F_2,E)\tilde{\in}(G_2,E)$.
Theorem:3.20
The following statements are equivalent for a soft topological space X:

1) X is soft πgr- normal.
2) For each soft closed set (A,E) and for each soft open set (U,E) containing (A,E), there exists a soft πgr-open set (V,E) containing (A,E) such that s-πgr-cl((V,E)) $\tilde{\subseteq}$ (U,E)
3) For each pair of disjoint soft closed sets (A,E) and (B,E), there exists soft πgr-open set (U,E) containing (A,E) such that s-πgr-cl((U,E))∩(B,E) = ϕ.

Proof:(1)⇒ (2): Let (A,E) be a soft closed set and (U,E) be a soft open set containing (A,E).
Then (A,E)∩(X−(U,E)) = ϕ and therefore they are disjoint soft closed sets in X.
Since X is soft πgr-normal, there exists disjoint soft πgr-open sets (V,E) and (W,E) such that (A,E) $\tilde{\subseteq}$ (V,E), X−(U,E) $\tilde{\subseteq}$ (W,E). i.e. X−(W,E) $\tilde{\subseteq}$ (U,E).
Now, (V,E)∩(W,E) = ϕ, implies (V,E)⊂X −(W,E)
Therefore, s-πgr-cl((V,E)) $\tilde{\subseteq}$ s-πgr-cl(X−(W,E))=X−(W,E), because X−(W,E) is soft πgr-closed set.
Thus, (A,E) $\tilde{\subseteq}$ (V,E) $\tilde{\subseteq}$ s-πgr-cl((V,E)) $\tilde{\subseteq}$ X−(W,E) ⊂(U,E).
i.e. (A,E) $\tilde{\subseteq}$ (V,E) $\tilde{\subseteq}$ s-πgr -cl((V,E)) $\tilde{\subseteq}$ (U,E)
Hence (2) holds.

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(2)⇒(3): Let (A,E) and (B,E) be disjoint soft closed sets in X, then (A,E) $\tilde{\subseteq}$ X−(B,E) and X−(B,E) is a soft open set containing (A,E). By hypothesis, there exists a soft πgr-open set (U,E) such that (A,E) $\tilde{\subseteq}$ (U,E) and s-πgr-cl((U,E)) $\tilde{\subseteq}$ X−(B,E), which implies s-πgr-cl((U,E))∩(B,E)) = ϕ

(3)⇒(1): Let (A,E) and (B,E) be two disjoint soft closed sets in X. By hypothesis (3), there exists a soft πgr-open set (U,E) such that (A,E) $\tilde{\subseteq}$ (U,E) and s-πgr-cl((U,E))∩(B,E) = ϕ (or) (B,E) $\tilde{\subseteq}$ X − s-πgr-cl((U,E)).

Now, (U,E) and X− s-πgr-cl((U,E)) are disjoint soft πgr-open sets such that (A,E) $\tilde{\subseteq}$ (U,E) and (B,E) $\tilde{\subseteq}$ X − s-πgr-cl((U,E)).

Hence X is soft πgr-normal .

Theorem : 3.21

If $f:SS(X)_E \to SS(Y)_E$ is soft continuous , soft bijective, soft πgr-open function from a soft normal space X onto a space Y, then Y is soft πgr-normal.

Proof: Let (F,E) and (G,E) be disjoint soft closed sets in Y,

Since f is soft continuous and soft bijective, f^{-1}((F,E)) and f^{-1}((G,E)) are disjoint soft closed sets in X.

Now, X is soft normal, there exists disjoint soft open sets (U,E) and (V,E) such that f^{-1}((F,E)) ⊂ (U,E), f^{-1}((G,E))⊂(V,E).

i.e. (F,E)⊂f((U,E)), (G,E)⊂f((V,E)).

Since f is soft πgr-open function, f((U,E)) and f((V,E)) are soft πgr-open sets in Y and f is injective, f((U,E))∩f((V,E)) = f((U,E)∩(V,E)) =f(ϕ) =ϕ.Hence Y is soft πgr-Normal.

Theorem:3.22

A soft topological space (X,τ,E) is soft πgr-normal iff for any soft πgr-closed set (F,E) and soft πgr-open set (G,E) containing (F,E) , there exists a soft πgr-open set (I,E) such that (F,E) $\tilde{\subseteq}$ (I,E) and s-πgr-cl(I,E) $\tilde{\subseteq}$ (G,E).

Proof: Let (X,τ,E) be a soft πgr-normal space and (F,E) be a soft πgr-closed set and (G,E) be a soft πgr-open set containing (F,E).

⇒ (F,E) and (G,E)′ are disjoint soft πgr-closed sets.

Then there exists two disjoint soft πgr-open sets (K_1,E) , (K_2, E) such that (F,E) $\tilde{\subseteq}$ (K_1,E) and (G,E)′ $\tilde{\subseteq}$ (K_2,E).

Now, (K_1,E) $\tilde{\subseteq}$ $(K_2,E)'$.

⇒ s-πgr-cl(K_1,E) $\tilde{\subseteq}$ s-πgr-cl$(K_2,E)'$ = $(K_2,E)'$ --------(1)

Also, (G,E)′ $\tilde{\subseteq}$ (K_2,E) ⇒ $(K_2,E)'$ $\tilde{\subseteq}$ (G,E)

⇒ s-πgr-cl(K_1,E) $\tilde{\subseteq}$ (G,E), by (1)

Put (K_1,E) = (I,E), we get the required result.

Conversely, let (H_1,E) and (H_2,E) be any disjoint pair of soft πgr-closed soft sets.

⇒ (H_1,E) $\tilde{\subseteq}$ $(H_2,E)'$, then by hypothesis, there exists a soft πgr-open set (K,E) such that (H_1,E) $\tilde{\subseteq}$ (K,E) and s-πgr-cl(K,E) $\tilde{\subseteq}$ $(H_2,E)'$.

⇒ (H_2,E) $\tilde{\subseteq}$ (s-πgr-cl(K,E))′

⇒ (K,E) and (s-πgr-cl(K,E))′ are disjoint soft πgr-open soft sets such that (H_1,E) $\tilde{\subseteq}$ (K,E) and (H_2,E) $\tilde{\subseteq}$ (s-πgr-cl(K,E))′

Hence the proof.

Theorem:3.23

Let f: $SS(X)_A \to SS(Y)_B$ be a soft surjective function which is both soft πgr-irresolute and soft M-πgr-open , where (X,τ,A) and (Y,τ*,B) are soft topological spaces. If (X,τ,A) is soft πgr-normal, then (Y ,τ*,B) is also soft πgr-normal.

Proof: Let (G,B) and (H,B) be a pair of disjoint πgr-closed soft sets of (V,τ*,B). Since, the function f is soft πgr-irresolute , f^{-1}(G,B) and f^{-1}(H,B) are disjoint soft πgr-closed sets of (X,τ,A).Since (X,τ*,B) is soft πgr-normal, there exists disjoint πgr-open soft sets (C,A) and (D,A) such that f^{-1}(G,B) $\tilde{\subseteq}$ (C,A) and f^{-1}(H,B) $\tilde{\subseteq}$ (D,A). The above implies (G,B) $\tilde{\subseteq}$ f(C,A) and (H,B) $\tilde{\subseteq}$ f(D,A).

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Since f is soft M-πgr-open, f(C,A) and f(D,A) are disjoint soft πgr-open sets of (Y,τ*,B) containing (G,B) and (H,B) respectively. Hence (Y,τ*,B) is also a soft πgr-normal space.

Theorem :3.24

If $f:SS(X)_E \rightarrow SS(Y)_E$ is soft πgr-continuous, soft closed bijective map and Y is soft normal, then X is soft πgr-normal.

Proof: Let (F,E) and (G,E) be disjoint soft closed sets in Y, since f is soft closed bijection, f((F,E)) and f((G,E)) are disjoint soft closed sets in Y.

Now Y is soft normal, there exists disjoint soft open sets (A,E) and (B,E) such that f((F,E)) $\tilde{\subseteq}$ (A,E), f((G,E)) $\tilde{\subseteq}$ (B,E).

$\Rightarrow$ (F,E) $\tilde{\subseteq}$ f^{-1}((A,E)) & (G,E) $\tilde{\subseteq}$ f^{-1}((B,E)).

Since f is soft πgr-continuous, f^{-1}((A,E)) and f^{-1}((B,E)) are soft πgr-open sets in X.

Further, f^{-1}((A,E)) $\cap$ f^{-1}((B,E)) = f^{-1}((A,E) $\cap$ (B,E)) =ϕ.Hence X is soft πgr-Normal.

Theorem :3.25

If $f:SS(X)_E \rightarrow SS(Y)_E$ is soft continuous, soft bijective, soft M-πgr-open function from a soft πgr - normal space X onto a space Y, then Y is soft πgr-normal .

Proof: Let (F,E) and (G,E) be two disjoint soft closed sets in Y . Since f is soft continuous and soft bijective, f^{-1}((F,E)) and f^{-1}((G,E)) are disjoint soft closed sets in X. Now, X is soft πgr-normal, there exists soft πgr-open sets (U,E) and (V,E) such that f^{-1}((F,E)) $\tilde{\subseteq}$ (U,E) and f^{-1}((G,E)) $\tilde{\subseteq}$ (V,E)That is (F,E) $\tilde{\subseteq}$ f((U,E)) and (G,E) $\tilde{\subseteq}$ f((V,E)). Since f is soft M-πgr-open function , f((U,E)) and f((V,E)) are soft πgr-open sets in Y and f is soft bijective,

f((U,E)) $\cap$f((V,E)) =f((U,E)$\cap$(V,E))=f(ϕ) =ϕ .

Hence Y is soft πgr-normal.

Theorem:3.26

If $f:SS(X)_E \rightarrow SS(Y)_E$ is soft πgr-irresolute, soft regular closed injection, and Y is soft πgr - normal, then X is also soft πgr-normal.

Proof: Let (F,E) and (G,E) be disjoint closed sets in Y. Since f is soft regular closed injection, f((F,E)) and f((G,E)) are disjoint soft regular closed sets in Y.

Now Y is soft πgr -Normal, there exists disjoint soft πgr-open sets (A,E) and (B,E) such that f((F,E))$\subset$(A,E), f((G,E))$\subset$(B,E).

This implies (F,E) $\tilde{\subseteq}$ f^{-1}((A,E)) and (G,E) $\tilde{\subseteq}$ f^{-1}((B,E)).

Since f is soft πgr-irresolute, f^{-1}((A,E)) and f^{-1}((B,E)) are soft πgr-open sets in X.

Further, f^{-1}((A,E)) $\cap$ f^{-1}((B,E)) = f^{-1}((A,E)) $\cap$ (B,E)) =ϕ.

$\Rightarrow$ X is soft πgr-Normal.

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